Algebraic Topology for the Physical Layer

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The rise of automated methods for synthesizing communication signals may require new tools for analysis in order to keep pace. Algebraic topology offers insights and tools that aid in automatic signal processing and complement traditional methods as well as newer methods such as, deep learning.
Share how algebraic topology could be used for blind OFDM synchronization and demodulation as well as automatic modulation surveying. Show why unsupervised methods for analyzing physical layers protocols may be required as parts of the PHY-layer become automatically created using machine learning.
Motivation
$x(n) = x_A(n) + e(n)$
Challenges
Learning New Physical Layer Waveforms with Autoencoders

input_signal = Input(shape=(M,))

transmitter_0 = Dense(M, activation='relu')(input_signal)
transmitter_1 = Dense(N, activation='linear')(transmitter_0)

channel = NoiseModel(K)(transmitter_2)

receiver_0 = Dense(M, activation='relu')(channel)
receiver_1 = Dense(M, activation='softmax')(receiver_0)

autoencoder = Model(input_signal, receiver_1)
autoencoder.compile(loss='categorical_crossentropy')
Automatically Learned PHY-layer for NASA’s TDRS by Deepsig
Non-cooperative Synchronization and Demodulation for OFDM
Create a computer program to automatically demodulate preamble waveforms without making too many assumptions and indirectly creating a DSP expert system.

**Assumptions**

- Sampling Rate
- Use of N-point FFT/IFFT-based modulation
Some Synchronization Methods from Least to Most Assumptions

- Energy Detection
- Delayed Conjugate Multiply
- Standard Deviation of fixed-sized FFT
- Cyclostationarity
- Schmidl-Cox
- Cross Correlation
Some Challenges in Non-cooperative OFDM Synchronization

- Unknown data
- Unknown preamble structure
- Unknown subcarrier modulation
- Unknown symbol to bit mapping
- Unknown OFDM parameters
- Unknown Channel
Need to clean up the signal to recover parameters for demodulation, but to remove noise we did to know the structure of the signal.
Topology Background
The Infinite vs. The Finite

\[
\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots \quad R = 1
\]

\[
e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \quad R = \infty
\]

\[
\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \quad R = \infty
\]

\[
\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \quad R = \infty
\]

\[
\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots \quad R = 1
\]

\[
\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \quad R = 1
\]

\[
(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \ldots \quad R = 1
\]
Simplicial Complex
Chain Group and Boundary Homomorphism

Simplicial Complex

$X_3(G) \rightarrow C_3(X(G))$

$X_2(G) \rightarrow \partial_3 \rightarrow C_2(X(G)) \ni$

$X_1(G) \rightarrow \partial_2 \rightarrow C_1(X(G)) \ni$

$X_0(G) \rightarrow \partial_1 \rightarrow C_0(X(G)) \ni 0$

Chain Complex $C_*(X(G))$
From Data to Simplicial Complex
Mapper

Point cloud → Z → Bin by intervals of Z → Cluster points → Mapper graph
Motivations

get a higher-level understanding of the structure of data

Mapper Algorithm

exhibit relations between clusters, variables, etc.

avoid paying the algorithmic price of persistence

visualize topology on the data directly

principle: summarize the topological structure of a map $f : X \rightarrow \mathbb{R}$ through a graph

Image source: http://www.enseignement.polytechnique.fr/informatique/INF563/
Unsupervised Machine Learning

Supervised

Unsupervised

Reinforcement
Results with and without Topology

Uniform Manifold Approximation and Projection

t-distributed Stochastic Neighbor Embedding
Uniform Manifold Approximation and Projection by McInnes
Results for Modulation Embedding
Uniform Manifold Approximation and Projection
Conclusion
Thank you!

https://www.shapeways.com/product/6CJQ9GXWV/topology-joke

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References III


