A decade of gr-specest

Free Spectral Estimation!

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What is Estimation (Theory)?

- An unambiguous, mathematically derived algorithm for determining a desired value from a set of noisy inputs
- No guesswork involved! “Guesstimates” are something else, not grounded in scientific rigour.
- Example 1: Determine constant the voltage on a line, but the measurement equipment is injecting white Gaussian noise, with an unknown, but constant and bounded variance

=> [do some math after establishing assumptions]

=> Best solution: Measure N times, average the results: \( \hat{u} = \frac{1}{N} \sum_{i=0}^{N-1} u_i \)
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- Example 2: How long will it take to complete a project, if there is exactly three tasks, and they can only be completed in order. Task 1 takes 3-6 days, Task 2 takes 2-5 days, Task 3 takes 1-3 days.

  => [Apply some common model]
  => 90% confidence of completion within 12.4 days

(*) PERT Model for SW project development was used here
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=> So, spectral estimation is the mathematical derivation of an estimate for “the spectrum” based on noisy measurements.
What is a spectrum?

- Come to think of it, what is a spectrum? What’s the most spectrum-esque picture here?

=> Understanding spectral analysis is useless without first defining “spectrum”
Non-Parametric Spectral Estimation
Estimating the Power Spectrum Density

- PSD as expected value of the truncated Fourier transform:

\[ S_{xx}(\omega) = \lim_{T \to \infty} \mathbb{E} \left[ |\hat{x}(\omega)|^2 \right] \]

Where

\[ \hat{x}(\omega) = \frac{1}{\sqrt{T}} \int_0^T x(t)e^{-i\omega t} \, dt \]

- “Average time-limited Fourier transform”

- PSD as Fourier transform of the autocorrelation function of the underlying stochastic process:

\[ S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-i\omega \tau} \, d\tau \]

where:

\[ R_{xx}(\tau) = \mathbb{E}[X(t)^* X(t + \tau)] \]

Notation and Definitions: Wikipedia

We know how to do time-limited Fourier transforms of discrete-time signals really fast, don’t we!

This path leads to correlogram-based methods (not further discussed here)
Welch’s Method

- Split signal into $L$ segments of length $M$ samples, overlapping by $D$ samples
- Multiply every segment by a window function (“modify the segment”)
- Calculate the FFT of every modified segment
- Calculate the magnitude-squared of every FFT output (“modified periodogram”)
- Calculate bin-wise average of periodograms
- For continuous updates, drop last periodogram and acquire new periodogram from new samples
welch_spectrum.grc
What is the “QT Frequency Sink”? 

- Less sophisticated little brother of the Welch estimator:
- No overlap
- Samples get dropped to accommodate QT Update Interval
- Averaging is done using single-pole IIR
- No power level normalization
- “Dropped some rigour for performance & convenience”
Practical Considerations

- Window Function: Affects sidelobes and scalloping loss
- FFT length: Affects FFT gain
- $\Rightarrow$ Power peak values are a function of both frequency, and the Welch configuration parameters!

Four different power levels???
Parametric Spectral Estimation
Sinusoidal Detection

- What if I’m only finding sinusoids in noise?
  Signal model:
  
  \[ y(k) = \sum_{i=0}^{n} \alpha_i e^{j(\omega_i k + \phi_i)} + z(k) \]

- Then all I’m looking for are parameters of my model
  - “What are the omegas?”

- Solution: Subspace-based, algebraic methods
- MUSIC, ESPRIT are the most commonly cited variations of these methods
- These algorithms can be used as efficient direction-finding algorithms

DEMO
esprit_freqest.grc and esprit_pspectrum.grc
ESPRIT and MUSIC

- We’ll skip the math to avoid incurring the wrath of the MC
- Pseudospectrum vs. Frequency Estimation:
  - We either immediately identify the parameters, and that’s it
  - Or we derive a spectral plot out of our method
- The model needs to be well defined: We need to know the number of sinusoids for a good estimate
- The input for MUSIC and ESPRIT is an estimate of the covariance matrix, which improves with more averaging
- The dimensionality of the covariance matrix is an algorithm parameter, it needs to be greater than the number of sinusoids
AR/MA Signal Models

- What if my signal is modeled by an LTI system? Maybe it’s a filter?
- How about I directly try and estimate the filter, instead of the entire signal?

**Common algorithm: Burg’s Method**
- Other algorithms: Yule-Walker Method, Fast (modified) Covariance Method
- Other underlying principles: Wiener-Hopf Equation, Levinson-Durbin Recursion
  PARCOR-Coefficients
burg.grc
gr-speceest’s nice little history
gr-spececest’s origins

- New challenge: Germany switched to a Bachelor’s / Master’s system
- In our case, students need to do Bachelor’s-Level graduation projects
- What can they do?
  - Need solid practical experience (might go look for a job)
  - Need solid theoretical foundations (might get a Master’s or Dr.-Ing.)
  - Should be fun
- Solution: Throw the students at GNU Radio, but let them figure out something with a good theoretical background
  - Easy way to get people in the community
  - Students had relevant open source code out there for employers who care
gr-specest on the webs

- We quickly disseminated and praised gr-specest everywhere, put it on CGRAN
- Students could point show their moms they were famous on the internet

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[Discuss-gnuradio] Spectral Estimation and Compressive Sensing

From: Martin Braun  
Subject: [Discuss-gnuradio] Spectral Estimation and Compressive Sensing  
Date: Thu, 5 Mar 2009 11:47:17 +0100  
User-agent: Mutt/1.5.17 (2007-11-01)

Hi List,

I am happy to say that we from the INT have managed to merge some of our research with GNU Radio development and have released some code on CGRAN. There are two new projects:

1) Spectral Estimation Toolbox

This project aims to enhance GNU Radio with 'proper' spectral estimation routines; so far it only includes Welch's method as a hierarchical block.
List of all gr-specest algorithms

- Welch’s Method (MA and Single-Pole)
- MUSIC (Root-MUSIC, MUSIC Pseudospectrum)
- ESPRIT (Frequency Estimator, Pseudospectrum)
- Thomson’s Multitaper Method (MTM)
- Burg’s Method
- Fast (modified) covariance method
- FFT Accumulation Method (Cyclostationary Processing, GUI version not yet ported to 3.8)

- Utilities: Vector Moving Average, Vector Reciprocal (1/x), Stream to Vector Overlap, Pad Vector
What else came out of gr-specest?

- A group of students benchmarked various math libraries (in 2010) and found that Fortran90 beat the competition -> So of course we now have Fortran in GNU Radio
- It started making us frustrated with creating OOTs. A few OOTs later, modtool was born.
- We learned there were more ways to capitalize on students’ work

```fortran
SUBROUTINE ZESPRIT(SAMPLES, LSAMPLES, N, M, OMEGAS)
  IMPLICIT NONE
  INTEGER :: LSAMPLES, N, M, I
  COMPLEX*16 :: SAMPLES(LSAMPLES), R(M,M), EV(N)
  DOUBLE PRECISION :: OMEGAS(N)

  CALL ZCORREST(SAMPLES, LSAMPLES, M, R)
  CALL ZESPRIT_COMMON(R, N, M, EV)

  DO I = 1, N
    OMEGAS(I) = ATAN2(AIMAG(EV(I)), REAL(EV(I)))
  END DO

END SUBROUTINE ZESPRIT
```